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Correspondence between the 3-point BMN correlators and the 3-string vertex on the pp-wave

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Abstract: The PP-wave/SYM proposal in its original form emphasizes a duality relation between the masses of the string states and the anomalous dimensions of the corresponding BMN operators in gauge theory, the *mass-dimension* type duality. In this paper, we give evidence in favour of another duality relation of the *vertex-correlator* type, which relates the coefficients of 3-point correlators of BMN operators in gauge theory to 3-string vertices in lightcone string field theory in the pp-wave background. We verify that all the available field theory results in the literature, as well as the newly obtained ones, for the 3-point functions are successfully reproduced from our proposal.

Keywords: Duality in Gauge Field Theories, Penrose limit and pp-wave background, Supersymmetry and Duality.

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1. Introduction

In this paper, we continue the study of the correspondence [1]–[3] between 3-point functions of BMN operators in $\mathcal{N} = 4$ Yang-Mills theory and 3-string interactions in the pp-wave lightcone string field theory.

The pp-wave/SYM proposal in its original form [4] emphasizes a duality relation between the anomalous dimensions of the BMN operators and the masses of the corresponding string states. Much progress had been made in verifying this relation in the planar limit of SYM perturbation theory [4]–[6], and at the nonplanar genus-one level [1] and [7]–[9] also incorporating important effects due to mixing of the planar BMN operators. Further investigations of the BMN sector in SYM were carried out in [10]–[17].

The *mass-dimension* type duality relation was clarified and extended in [18]–[20] (see also [21, 22]) where it was expressed in the form

$$H_{\text{string}} = H_{\text{SYM}} - J\,. \tag{1.1}$$

Here H_{string} is the full string field theory hamiltonian, and $H_{\text{SYM}} - J = \Delta - J$ is the gauge theory hamiltonian (the conformal dimension) minus the R-charge. The relation eq. (1.1) is expected to be exact and hold to all orders in the two free parameters of the theory, g_2 and λ'.

However, one can argue that if the the relation eq. (1.1) was all that there is in the pp-wave/SYM correspondence, this would not add much to our understanding of neither the interactions of the massive modes in string theory, nor to the gauge theory dynamics in the large N double scaling limit. It would also be rather unsatisfactory aesthetically. Recall that in the original AdS/CFT proposal, in addition to the relation between the masses of supergravity states (and their KK towers) and the conformal dimensions of the dual operators in SYM, one could also compare directly the correlation functions in gauge theory with the bulk interaction vertices [23, 24] using the bulk-boundary propagators. Since the pp-wave/CFT correspondence can be considered as a particular limit of the AdS/CFT correspondence, it is natural to suspect that similar *vertex–correlator* type duality relation will hold in the pp-wave/SYM correspondence. Since the attempts so far [25] of uncovering similar to the AdS/CFT holographic relation turned out to be extremely difficult and somewhat unfruitful, there is a need for a different route to establish a dynamical vertex-correlator type pp-wave duality (if any). In this paper we will use the large body of recently derived detailed field theory results for BMN correlation functions as the 'experimental data' for building up a theoretical model of such a relation.

An important observation that the 3-point function in a conformal field theory takes a universal form was put to use in [1, 3]. In conformal theory, the two- and three-point functions of conformal primary operators are completely determined by conformal invariance of the theory. One can always choose a basis of primary operators such that the two-point functions take the canonical form:

$$\langle \mathcal{O}_I(x_1)\bar{\mathcal{O}}_J(x_2)\rangle = \frac{\delta_{IJ}}{(4\pi^2 x_{12}^2)^{\Delta_I}}, \tag{1.2}$$

and all the nontrivial information of the three-point function is contained in the x-independent coefficient C_{123}:

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\bar{\mathcal{O}}_3(x_3)\rangle = \frac{C_{123}}{(4\pi^2 x_{12}^2)^{\frac{\Delta_1+\Delta_2-\Delta_3}{2}}(4\pi^2 x_{13}^2)^{\frac{\Delta_1+\Delta_3-\Delta_2}{2}}(4\pi^2 x_{23}^2)^{\frac{\Delta_2+\Delta_3-\Delta_1}{2}}}\,, \tag{1.3}$$

where $x_{12}^2 := (x_1 - x_2)^2$. Since the form of the x-dependence of conformal 3-point functions is universal, one can identify the coefficient C_{123} with a 'coupling constant' of the three BMN states in SYM. It is then natural to expect that C_{123} is related to the interaction of the corresponding three string states in the pp-wave background.[1] Using the operator

[1] There is also a more technical reason for this relation: we will show in section 3 that all the available SYM results for $\mu(\Delta_1 + \Delta_2 - \Delta_3)\, C_{123}$ can be expressed entirely in terms of the natural pp-wave string theory quantities, such as Neumann matrices, oscillation frequencies etc.

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product expansion, this 3-point relation will then serve as the building block for a string interpretation of n-point BMN correlators [12] in short distance limits.

A few general remarks are in order:

1. We note that it is an essential part of our proposal to use on the SYM side the BMN operators defined in such a way that they do not mix with each other (i.e. have definite scaling dimensions Δ) and which are conformal primary operators. The BMN operators defined in this way will be called the Δ-BMN operators. Conformal invariance of the $\mathcal{N}=4$ theory then implies that the 2-point correlators of these Δ-BMN operators are canonically normalized, and the 3-point functions take the simple form eq. (1.3).

2. The relation (1.1) can be understood as the equivalence of the spectra of the operators or in a stronger form, as an operator equation. To establish the latter, one would have to first establish an isomorphism of the field theory and string theory Hilbert spaces, and then compare the matrix elements of the operators in eq. (1.1). This point of view was adopted in, e.g. [19, 20], where certain modified BMN operator bases were considered. Each of the two bases of [19] and [20] was reported to be isomorphic to the Hilbert space of *bare string states*, i.e. the basis which one uses to write down the tree-level 3-string vertex. By construction, the bases of [19, 20] were not the eigenstates of Δ, and hence different from the Δ-BMN basis which we use here. Because of this, each of the bases of [19, 20] was made orthonormal only at the *free* field theory level. However at the interacting level ($\lambda' \neq 0$) the 2-point functions of the operators in [19, 20] will contain a non-universal logarithmic coordinate dependence. It is not clear to us how to remove this dependence from the 2-point functions and to define a coordinate-independent overlaps, unless one is using the Δ-BMN basis, where the coordinate dependence is universal, i.e. dictated by eq. (1.2).

3. In this paper we are not attempting to construct the isomorphism between the states in string theory and in the BMN sector of SYM. For example, our Δ-BMN basis[2] is not isomorphic to the natural basis of bare string states. Our proposal is, instead, to relate the naturally defined in conformal field theory coefficient C_{123} with the tree level string interaction of bare string states. For this purpose, the Δ-BMN basis is unique, as only for such a basis one can write down eq. (1.3) and the coefficient C_{123} is defined.

The paper is organized as follows. In section 2, we will examine and elaborate on a specific vertex-correlator duality relation (eq. (2.1) below) originally proposed in [1]. In section 3, we will subject this proposal to (twelve) tests. We will show that all the results for field theory 3-point functions that are available in the literature up to date, including BMN operators with 2 impurities, [3, 8] and with 3 impurities, [26], can be precisely reproduced on the string theory side with a specific choice of the string theory prefactor P on the right

[2]It can be used, however, and is well-suited for calculating the spectum of (1.1) on the SYM side.

hand side of eq. (2.13). We emphasize that this matching is nontrivial even though within our approach the choice of the prefactor is “phenomenological”. A first principles derivation of the string field theory prefactor is highly desirable, but not yet available, [27, 28], inspite of a recent progress [29] and much work on the construction of the 3-string vertices in the pp-wave lightcone string field theory [30]–[34] and [27]–[29].

In this paper we are not concerned with fermionic BMN operators in gauge theory, hence we are not probing the fermionic structure of the 3-string vertex. Our prefactor is an effective bosonic part of the full prefactor, it is clearly $\mathbb{Z}_2$ invariant, but we cannot study the full (super)-symmetry of the vertex in the pp-wave background without including fermions.

2. The correspondence between field theory 3-point function and 3-string vertex

The idea is to compare directly the 3-strings interaction amplitude with the field theory structure coefficients via [1]

$$\mu(\Delta_1 + \Delta_2 - \Delta_3)\, C_{123} = \langle\Phi_1|\langle\Phi_2|\langle\Phi_3|H_3\rangle\,. \tag{2.1}$$

Here $\langle\Phi_1|\langle\Phi_2|\langle\Phi_3|H_3\rangle$ is the three-string scattering amplitude in the string field theory and $|H_3\rangle$ is the lightcone three-string vertex. C_{123} is the three-point function coefficient in (1.3) of the corresponding BMN operators. We propose that equation (2.1) is valid to all orders in perturbation theory in the effective gauge coupling λ' of the BMN sector,

$$\lambda' = \frac{g_{\rm YM}^2 N}{J^2} = \frac{1}{(\mu p^+ \alpha')^2} \tag{2.2}$$

and at the leading order in the field theory genus counting parameter

$$g_2 := \frac{J^2}{N} = 4\pi g_s (\mu p^+ \alpha')^2\,. \tag{2.3}$$

To proceed, we need to specify the expression for the 3-string vertex $|H_3\rangle$. The 3-string vertex can be represented as a ket-state in the tensor product of the three string Fock spaces. It has the form

$$|H_3\rangle = \mathsf{P}|V_F\rangle|V_B\rangle\delta\left(\sum_{\rm r=1}^{3}\alpha_{\rm r}\right), \tag{2.4}$$

where the kets $|V_B\rangle$ and $|V_F\rangle$ are constructed to satisfy the bosonic and fermionic kinematic symmetries and $\alpha_{\rm r}$ are defined in eq. (A.1) in the appendix. The bosonic factor $|V_B\rangle$ is given by

$$|V_B\rangle = \exp\left(\frac{1}{2}\sum_{\rm r,s=1}^{3}\sum_{m,n=-\infty}^{\infty}\sum_{I=1}^{8}\alpha_m^{{\rm r}\,I\dagger}\hat N^{\rm rs}_{mn}\alpha_n^{{\rm s}\,I\dagger}\right)|0\rangle_{123}\,, \tag{2.5}$$

where the $\hat N^{\rm rs}_{mn}$ are the Neumann matrices in the BMN-basis of string oscillators (as defined in eq. (A.6)). To simplify our notation in what follows we suppress the explicit sum over the

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I indices. The complete perturbative expansion of the Neumann matrices in the pp-wave background in the vicinity of $\mu = \infty$, was recently constructed in [29].[3] The prefactor P is a polynomial in the bosonic and fermionic oscillators and should be determined from imposing the remaining symmetries of the pp-wave background. The fermionic factor $|V_F\rangle$ is not going to be relevant for the present paper where only external *bosonic* string states are considered.

The construction of the 3-string vertex has been considered in [30, 31, 33, 34], however as emphasized in [27, 28], string theory in the pp-wave background must respect the full symmetry of the background, including the bosonic symmetry $\mathrm{SO}(4)\times\mathrm{SO}(4)\times\mathbb{Z}_2$, where the $\mathbb{Z}_2$ exchanges the first SO(4) with the second SO(4). It turns out that the string interactions constructed in [30, 31, 33, 34] do not respect the $\mathbb{Z}_2$ symmetry and so cannot fully describe the string interaction in the pp-wave background. Implementation of the $\mathbb{Z}_2$ symmetry at the level of the fermionic overlap $|V_F\rangle$ has been performed in [27, 28]. Explicit expression of $|V_F\rangle$ is given in [28, eqn.(16)]. Based on this starting point, one can at least in principle construct the prefactor P by imposing on the vertex dynamical (super)symmetries of the background. However the algebra is quite involved [35] and the explicit form for the prefactor has not yet been determined from the first principles.

In this paper we take a different approach and instead of deriving the prefactor in string theory we propose a simple ansatz for the bosonic part of the prefactor P which is then subjected to numerous independent tests against all the available field theory expressions. Our ansatz for the relevant bosonic part of the prefactor is

$$\mathsf{P} = C_{\rm norm}(\mathsf{P}_I + \mathsf{P}_{II}) \,, \tag{2.6}$$

where[4]

$$\mathsf{P}_I = \sum_{\mathrm{r}=1}^{3}\sum_{m=-\infty}^{+\infty} \frac{\omega_{\mathrm{r}m}}{\alpha_{\mathrm{r}}}\,\alpha_m^{\mathrm{r}\,I\dagger}\alpha_m^{\mathrm{r}\,I} \,, \tag{2.7}$$

and

$$\mathsf{P}_{II} = \frac{1}{2}\sum_{\mathrm{r,s}=1}^{3}\sum_{m,n>0} \frac{\omega_{\mathrm{r}m}}{\alpha_{\mathrm{r}}}\,(\hat N^{\mathrm{rs}}_{m-n} - \hat N^{\mathrm{rs}}_{mn})(\alpha_m^{\mathrm{r}\,I\dagger}\alpha_n^{\mathrm{s}\,I\dagger} + \alpha_{-m}^{\mathrm{r}\,I\dagger}\alpha_{-n}^{\mathrm{s}\,I\dagger} - \alpha_m^{\mathrm{r}\,I\dagger}\alpha_{-n}^{\mathrm{s}\,I\dagger} - \alpha_{-m}^{\mathrm{r}\,I\dagger}\alpha_n^{\mathrm{s}\,I\dagger}) \,. \tag{2.8}$$

Apart from an overall numerical coefficient,[5]

$$C_{\rm norm} = g_2\frac{\sqrt{y(1-y)}}{\sqrt{J}} = C^{\rm vac}_{123} \,, \tag{2.9}$$

our prefactor eq. (2.6) is the sum of two contributions, P_I and P_{II}.

[3] We refer the reader to the appendix for some useful properties of the perturbative Neumann matrices, relations between different string-oscillator bases, and the comparison with other results in the literature.

[4] Throughout the paper we are using the usual definitions for the SFT quantities in the pp-wave background such as $\omega_{\mathrm{r}m}$, α_{r} and μ, which are summarized in the appendix.

[5] Which turns out to be precisely equal to the structure constant $C^{\rm vac}_{123}$ of the "vacuum" BMN operators $\mathcal{O}^J_{\rm vac} := (JN^J)^{-1/2}\,\mathrm{tr}(Z^J)$.

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The first contribution P_I, given by eq. (2.7), is simply the difference between the light-cone energies of the string states[6] or, equivalently, μ times the difference of the scaling dimensions of the incoming and outgoing BMN operators, $\Delta_1 + \Delta_2 - \Delta_3$. The second contribution P_{II}, given in eq. (2.8), is also bilinear in string oscillators, but involves creation operators only. For future reference we also note that the sum eq. (2.8) excludes the supergravity modes $m, n = 0$.

It is worthwhile to note that our ansatz for the prefactor, (2.7) and eq. (2.8), takes a remarkably simple form when expressed in terms of the original SFT a-oscillator basis [7]

$$\mathsf{P}_I = \sum_{\mathrm{r}=1}^{3} \left(\sum_{m>0} \frac{\omega_{\mathrm{r}m}}{\alpha_{\mathrm{r}}} \left(a_m^{\mathrm{r}\,I\dagger} a_m^{\mathrm{r}\,I} + a_{-m}^{\mathrm{r}\,I\dagger} a_{-m}^{\mathrm{r}\,I} \right) + \mu \,\mathrm{sign}(\alpha_{\mathrm{r}}) a_0^{\mathrm{r}\,I\dagger} a_0^{\mathrm{r}\,I} \right), \tag{2.10}$$

$$\mathsf{P}_{II} = -\sum_{\mathrm{r}=1}^{3} \sum_{m>0} \frac{\omega_{\mathrm{r}m}}{\alpha_{\mathrm{r}}} a_{-m}^{\mathrm{r}\,I\dagger} a_{-m}^{\mathrm{r}\,I} . \tag{2.11}$$

Hence the full prefactor is

$$\mathsf{P} = \sum_{\mathrm{r}=1}^{3} \left(\sum_{m>0} \frac{\omega_{\mathrm{r}m}}{\alpha_{\mathrm{r}}} a_m^{\mathrm{r}\,I\dagger} a_m^{\mathrm{r}\,I} + \mu \,\mathrm{sign}(\alpha_{\mathrm{r}}) a_0^{\mathrm{r}\,I\dagger} a_0^{\mathrm{r}\,I} \right). \tag{2.12}$$

This is different from the earlier proposals for the prefactor in [31, 33, 29]. Although the prefactor takes a simpler form in the SFT a-oscillator basis, we will continue using the prefactor in the BMN α-oscillator basis, (2.7) and (2.8), where the comparison with the gauge theory BMN correlators is more direct.

Our prefactor, and in particular the second term P_{II}, is constructed to reproduce a particular class of field theory results[8] for the 3-point functions. It will turn out that the relatively simple expressions for P_I and P_{II} will match with *all* of the available field theory results, thus passing numerous non-trivial tests detailed in section 3. We also find it encouraging that the coefficient matrix in front of the oscillator-bilinear in eq. (2.11) or eq. (2.8) is assembled directly from the Neumann matrices rather than being given by a generic function of m, n and r, s. Since the Neumann matrices are known [29] to all orders in the perturbative expansion in inverse powers of μ, our proposed correspondence provides an all-orders in λ' prediction for the 3-point BMN correlators in SYM.

To summarize: for the bosonic external string states $\langle \Phi_i |$ our proposed correspondence relation is

$$\mu(\Delta_1 + \Delta_2 - \Delta_3)\, C_{123} = \langle \Phi_1 | \langle \Phi_2 | \langle \Phi_3 | \,\mathsf{P} \exp\left(\frac{1}{2} \sum_{\mathrm{r,s}=1}^{3} \sum_{m,n=-\infty}^{\infty} \sum_{I=1}^{8} \alpha_m^{\mathrm{r}\,I\dagger} \hat{N}_{mn}^{\mathrm{rs}} \alpha_n^{\mathrm{s}\,I\dagger} \right) |0\rangle_{123} , \tag{2.13}$$

[6] In this paper we always assume that the outgoing string state and one of the incoming string states are excited states, i.e. have non-zero eigenvalues of the number operator $\alpha_m^{\dagger}\alpha_m$.

[7] To derive (2.11), we have used $\mathsf{P}_{II} = -\sum_{\mathrm{r,s}=1}^{3} \sum_{m,n>0} \frac{\omega_{\mathrm{r}m}}{\alpha_{\mathrm{r}}} N_{-m-n}^{\mathrm{rs}}\, a_{-m}^{\mathrm{r}\,I\dagger} a_{-n}^{\mathrm{s}\,I\dagger}$, which follows from (2.8) directly, the properties (A.14) and the fact that when acting on $|V_B\rangle$, a acts like a derivative with respect to $a^{\dagger}$.

[8] Namely the two expressions considered in subsections 3.3.1 and 3.3.2.

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where P is given by eq. (2.6) and $\hat{N}_{mn}$ are the perturbative Neumann matrices of [29] detailed in the appendix.

3. Tests of the proposal

As explained earlier, on the SYM side of our proposed correspondence we must use the Δ-BMN operator basis. For BMN operators with 2 scalar impurities this basis was constructed in [8] to order $g_2(\lambda')^0$ and $g_2{}^2(\lambda')^0$ and involves a linear combination of the original single-trace BMN operator and the double-trace (in general multi-trace) BMN operators.

3.1 SYM predictions

All the currently known SYM results for 3-point BMN correlators can be divided into two broad classes. The *first class* involves 1 general and 2 chiral Δ-BMN operators (i.e. 1 string and 2 supergravity states). Furthermore, no flavour-changing processes are allowed for the 3-point functions of the first class. The *second class* involves 2 general Δ-BMN operators (i.e. 2 string states and 1 supergravity state) with or without flavour changing, and the flavour changing 1 string $\to$ 2 supergravity processes.

The 3-point functions of the first class can be calculated directly with the original single-trace BMN operators since it is easy to check that the contributions coming from the double-trace operators vanish (at the first non-vanishing order in g_2). Three-point functions of the first class were calculated in [3] for 2 scalar impurities and, in the follow-up paper [26], for 3 scalar impurities at the leading non-trivial order in g_2. For the second class the contributions from the double-trace operators are important and one should use the Δ-BMN basis. The calculations of the 3-point functions in this basis with 2 scalar impurities were carried out in [8].

All the currently known SYM results for 3-point correlators with 2 scalar impurities can be summarized by the following expression for the 3-point function coefficients C_{123}:

$$C(k_n l_{-n},\, \mathrm{vac}|\, i_m j_{-m}) \,=\, C_{123}^{\mathrm{vac}} \frac{2\,\sin^2(\pi m y)}{y\,\pi^2(m^2-n^2/y^2)^2}\left(\delta_{i(k}\delta_{l)j}\; m^2 + \delta_{i[k}\delta_{l]j}\,\frac{mn}{y} + \frac{1}{4}\delta_{ij}\delta_{kl}\,\frac{n^2}{y^2}\right)\,, \tag{3.1}$$

$$C(k_0,\, l_0|\, i_m j_{-m}) \,=\, C_{123}^{\mathrm{vac}} \frac{2}{\sqrt{y(1-y)}}\left(\delta_{m,0}\, y - \frac{\sin^2(\pi m y)}{\pi^2 m^2}\right)\delta_{i(k}\delta_{l)j}\,, \tag{3.2}$$

where $C(1,2|3)$ is the coefficient for the 3-point function $\langle \mathcal{O}_1^{J_1}\mathcal{O}_2^{J_2}\bar{\mathcal{O}}_3^{J}\rangle$, $J=J_1+J_2$ and

$$y := \frac{J_1}{J} \tag{3.3}$$

is the R-charge ratio. Here the "string modes" m and n can be positive, negative or zero, impurities flavours i,j,k and l are arbitrary integers from the set $\{1,2,3,4\}$, and the symmetric traceless and the antisymmetric traceless combination of the two Kroneckers are defined as

$$\delta_{i(k}\delta_{l)j} = \frac{1}{2}(\delta_{ik}\delta_{lj}+\delta_{il}\delta_{kj}) - \frac{1}{4}\delta_{ij}\delta_{kl}\,, \qquad \delta_{i[k}\delta_{l]j} = \frac{1}{2}(\delta_{ik}\delta_{lj}-\delta_{il}\delta_{kj})\,. \tag{3.4}$$

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When $n = 0$ and $i, j = k, l$ or $i, j = l, k$, these 3-point functions are from the first class and were originally derived in [3]. Otherwise these results are from the second class and were derived in [8]. It is important to note that the calculations of [3, 8] were performed to order $(\lambda')^1$, incorporating the leading order in λ' anomalous dimensions of the Δ-BMN operators in eq. (1.3). However, the resulting expressions for the three-point coefficients C_{123} can be trusted only to order $(\lambda')^0$ [8]. This is because the yet unaccounted order $g_2(\lambda')^1$ corrections to the mixing matrices of the single- and the double-trace operators will affect the order $(\lambda')^1$ expressions for C_{123} (but not the logarithmic anomalous dimensions). Hence, the expressions on the right hand side of eq. (3.1), eq. (3.2) are given at order $(\lambda')^0$. We stress that they are different from the naive free field theory results as they already incorporate the operator mixing at order $g_2(\lambda')^0$.

To go beyond these SYM results involving the 3-point Δ-BMN functions with 2 scalar impurities one can consider, for example, BMN operators with 3 scalar impurities i, j, k with string oscillator numbers n_i, n_j, n_k satisfying the constraint $n_i + n_j + n_k = 0$,

$$\mathcal{O}_{i_{n_i} j_{n_j} k_{n_k}} = \tag{3.5}$$
$$= \frac{1}{J\sqrt{N^{J+3}}} \sum_{0\le a,b}^{a+b\le J} \left[\mathrm{tr}\left(Z^a \phi_j Z^b \phi_k Z^{J-a-b} \phi_i \right) q_j^a q_k^{a+b} + \mathrm{tr}\left(Z^a \phi_k Z^b \phi_j Z^{J-a-b} \phi_i \right) q_k^a q_j^{a+b} \right],$$

where $q_j = e^{2\pi i n_j/J}$ and $q_k = e^{2\pi i n_k/J}$ are the BMN phase-factors. Three-point functions involving of Δ-BMN operators with 3 scalar impurities were evaluated very recently and will be reported in detail in the follow-up paper [26]. A simple example of a more general construction in [26] involves a 3-point function of 1 string-state BMN operator $\bar{\mathcal{O}}_{i_{n_i} j_{n_j} k_{n_k}}$, and 2 supergravity-state operators, $\mathcal{O}_{i_0, j_0, k_0}$ and $\mathcal{O}_{\mathrm{vac}}$, which is of conformal form eq. (1.3), giving the result for the coefficient:

$$C(1_0 2_0 3_0,\, \mathrm{vac}|\, 1_{n_1} 2_{n_2} 3_{n_3}) = -C_{123}^{\mathrm{vac}} \frac{\sin(\pi y n_1) \sin(\pi y n_2) \sin(\pi y n_3)}{y^{3/2} \pi^3\, n_1 n_2 n_3}\,. \tag{3.6}$$

Another simple example is a 3-point function of the same 1 string-state BMN operator $\bar{\mathcal{O}}_{i_{n_i} j_{n_j} k_{n_k}}$, and 2 lower-impurity supergravity-state operators, $\mathcal{O}_{i_0, j_0}$ and $\mathcal{O}_{k_0}$,

$$C(1_0 2_0,\, 3_0|\, 1_{n_1} 2_{n_2} 3_{n_3}) = C_{123}^{\mathrm{vac}} \frac{\sin(\pi y n_1) \sin(\pi y n_2) \sin(\pi y n_3)}{y\sqrt{1-y}\pi^3\, n_1 n_2 n_3}\,. \tag{3.7}$$

Having collected the field theory results above, we are now ready to perform explicit tests of our proposed correspondence. In doing so we will be testing the duality relation (2.13) itself, our ansatz for the prefactor eq. (2.6)–eq. (2.9) and the expressions for the Neumann matrices [29] from the appendix.

We will now split the corresponding string interactions according to the quantum numbers of the external states. From now on we will always assume that the different impurity indices i and j always take different values, $i \neq j$ and the same impurities will be denoted explicitly as i and i. Also the oscillator modes m and n will always be positive, the negative modes will be denoted as $-m$ and $-n$, and the supergravity states as 0.

3.2 Two supergravity states and one string state with two impurities

On the SYM side the 'supergravity' mode $n=0$ and at large μ we have $\mu(\Delta_1+\Delta_2-\Delta_3) = -m^2/\mu$. On the SFT side for two supergravity and one string state process P_{II} cannot contribute since it includes the contributions only from non-zero modes. Hence, only P_I contributes which is a diagonal operator with the eigenvalues

$$\mathsf{P}_I = \mu(\Delta_1+\Delta_2-\Delta_3) = -\frac{m^2}{\mu}\,. \tag{3.8}$$

3.2.1 $\mathbf{i_0 j_0 + vac \to i_m j_{-m}}$

Here there are 2 supergravity states with no flavour-changing, hence the process belongs to the first class.h The SYM prediction eq. (3.1) is

$$\mu(\Delta_1+\Delta_2-\Delta_3)C_{123} = -C_{123}^{\rm vac}\,\frac{\sin^2(\pi m y)}{y\,\pi^2\mu}\,. \tag{3.9}$$

The product of the external string states is

$$_{123}\langle 0|\alpha_m^{3i}\alpha_{-m}^{3j}\alpha_0^{1i}\alpha_0^{1j}\,, \tag{3.10}$$

and the relevant part of the exponent in eq. (A.15) reads

$$\exp[\hat{N}_{m0}^{31}(\alpha_m^{3i^\dagger}\alpha_0^{1i^\dagger}+\alpha_{-m}^{3j^\dagger}\alpha_0^{1j^\dagger})]\,. \tag{3.11}$$

The resulting string field theory expression, cf eq. (2.13), is

$$-\frac{m^2}{\mu}\,C_{123}^{\rm vac}\,\hat{N}_{m0}^{31}\,\hat{N}_{m0}^{31} = \frac{m^2}{\mu}\,C_{123}^{\rm vac}\,\frac{\sin^2(\pi m y)}{\pi^2 m^2 y}\,, \tag{3.12}$$

which reproduces the gauge theory result eq. (3.9).

3.2.2 $\mathbf{i_0 + j_0 \to i_m j_{-m}}$

This is also the first class process. The gauge theory expression eq. (3.2) gives

$$\mu(\Delta_1+\Delta_2-\Delta_3)C_{123} = C_{123}^{\rm vac}\,\frac{\sin^2(\pi m y)}{\sqrt{y(1-y)}\,\pi^2\mu}\,. \tag{3.13}$$

The product of the external string states is now

$$_{123}\langle 0|\alpha_m^{3i}\alpha_{-m}^{3j}\alpha_0^{1i}\alpha_0^{2j}\,, \tag{3.14}$$

and the relevant part of the exponent is again eq. (3.11). This gives the string prediction

$$-\frac{m^2}{\mu}\,C_{123}^{\rm vac}\,\hat{N}_{m0}^{32}\,\hat{N}_{m0}^{31} = \frac{m^2}{\mu}\,C_{123}^{\rm vac}\left(\frac{\sin(\pi m y)}{\pi m\sqrt{1-y}}\right)\left(-\frac{\sin(\pi m y)}{\pi m\sqrt{y}}\right), \tag{3.15}$$

which is in agreement with the SYM expression eq. (3.13).

3.2.3 $\mathbf{j_0 j_0 + vac \to i_m i_{-m}}$

This is a flavour changing process since $i \neq j$ and it belongs to the second class. The SYM prediction eq. (3.1) is

$$\mu(\Delta_1 + \Delta_2 - \Delta_3)C_{123} = \frac{1}{2}\, C_{123}^{\rm vac}\, \frac{\sin^2(\pi m y)}{y\,\pi^2\mu}\,. \tag{3.16}$$

The product of the external string states is

$$_{123}\langle 0|\alpha_m^{3i}\alpha_{-m}^{3i}\alpha_0^{1j}\,\alpha_0^{1j}\,. \tag{3.17}$$

Since this is a flavour changing process, the leading order in $1/\mu$ contribution comes from

$$\mathsf{P}_{II} = -\frac{\omega_{3m}}{\alpha_3}\,(\hat{N}^{33}_{m-m} - \hat{N}^{33}_{mm})(\alpha_m^{3i\dagger}\,\alpha_{-m}^{3i\dagger}) = \frac{2}{\pi}\sin^2(\pi m y)(\alpha_m^{3i\dagger}\,\alpha_{-m}^{3i\dagger})\,, \tag{3.18}$$

while P_I does not contribute. In deriving eq. (3.18) we have used the fact that

$$\hat{N}^{33}_{m-m} - \hat{N}^{33}_{mm} = N^{33}_{-m-m} = \frac{2}{\mu\pi}\sin^2(\pi m y) \propto \frac{1}{\mu}\,, \tag{3.19}$$

and

$$\hat{N}^{\rm rr}_{m-m} - \hat{N}^{\rm rr}_{mm} = N^{\rm rr}_{-m-m} = \mathcal{O}\left(\frac{1}{\mu^3}\right), \quad \text{for} \quad \mathrm{r} = 1,2\,. \tag{3.20}$$

From the exponent eq. (A.15) in eq. (2.13) we get the factor of

$$\hat{N}^{11}_{00}\,\alpha_0^{1j\dagger}\,\alpha_0^{1j\dagger} = \frac{1}{\mu\,4\pi y}\alpha_0^{1j\dagger}\,\alpha_0^{1j\dagger}\,. \tag{3.21}$$

Substituting these expressions into eq. (2.13) we get the string theory prediction

$$\frac{1}{2}\, C_{123}^{\rm vac}\, \frac{\sin^2(\pi m y)}{y\,\pi^2\mu}\,, \tag{3.22}$$

which is precisely the SYM result eq. (3.16).

3.2.4 $\mathbf{i_0 i_0 + vac \to i_m i_{-m}}$

The SYM result eq. (3.1) is

$$\mu(\Delta_1 + \Delta_2 - \Delta_3)C_{123} = -\frac{3}{2}\, C_{123}^{\rm vac}\, \frac{\sin^2(\pi m y)}{y\,\pi^2\mu}\,. \tag{3.23}$$

The string theory prediction is equal twice the contribution from the subsection 3.2.1 plus the contribution from the subsection 3.2.3, which amounts precisely to the right hand side of eq. (3.23).

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3.2.5 $\mathbf{j_0 + j_0 \to i_m i_{-m}}$

The SYM prediction eq. (3.2) is

$$\mu(\Delta_1 + \Delta_2 - \Delta_3)C_{123} = -\frac{1}{2}\, C_{123}^{\rm vac}\, \frac{\sin^2(\pi m y)}{\sqrt{y(1-y)}\,\pi^2\mu}\,. \tag{3.24}$$

The product of the external string states is

$${}_{123}\langle 0|\alpha_m^{3i}\alpha_{-m}^{3i}\alpha_0^{1j}\,\alpha_0^{2j}\,. \tag{3.25}$$

This is a flavour changing process, P_I does not contribute and the leading order in $1/\mu$ contribution comes from P_{II} given by eq. (3.18). From the exponent eq. (A.15) in eq. (2.13) we get the factor of

$$\hat{N}_{00}^{21}\,\alpha_0^{1j\dagger}\,\alpha_0^{1j\dagger} = -\frac{1}{\mu 4\pi\sqrt{y(1-y)}}\alpha_0^{1j\dagger}\,\alpha_0^{1j\dagger}\,. \tag{3.26}$$

Substituting these expressions into eq. (2.13) we get the string theory prediction

$$-\frac{1}{2}\,C_{123}^{\rm vac}\,\frac{\sin^2(\pi m y)}{\sqrt{y(1-y)}\,\pi^2\mu}\,, \tag{3.27}$$

which is precisely the SYM result eq. (3.24).

3.2.6 $\mathbf{i_0 + i_0 \to i_m i_{-m}}$

The SYM prediction eq. (3.2) is

$$\mu(\Delta_1 + \Delta_2 - \Delta_3)C_{123} = \frac{3}{2}\, C_{123}^{\rm vac}\, \frac{\sin^2(\pi m y)}{\sqrt{y(1-y)}\,\pi^2\mu}\,. \tag{3.28}$$

It is easy to see that the corresponding string prediction is equal to twice the string prediction of eq. (3.13) plus the string prediction of eq. (3.27). This is in perfect agreement with the SYM result eq. (3.28).

3.3 Two string states with two impurities

On the SYM side at large μ we have $\mu(\Delta_1+\Delta_2-\Delta_3) = -(m^2 - n^2/y^2)/\mu$. On the SFT side this is matched by $\mathsf{P}_I = \mu(\Delta_1+\Delta_2-\Delta_3) = -(m^2-n^2/y^2)/\mu$, but now also the second term in the prefactor, P_{II}, has to be taken into account.

3.3.1 $\mathbf{i_n j_{-n} + vac \to i_m j_{-m}}$

The SYM prediction eq. (3.1) is

$$\mu(\Delta_1 + \Delta_2 - \Delta_3)C_{123} = -C_{123}^{\rm vac}\,\frac{\sin^2(\pi m y)}{y\,\pi^2\mu}\,\frac{m^2 + mn/y}{m^2 - n^2/y^2}. \tag{3.29}$$

The external string states are ${}_{123}\langle 0|\alpha_m^{3i}\alpha_{-m}^{3j}\alpha_n^{1i}\alpha_{-n}^{1j}$ and the contributing terms in P_{II} take form, cf eq. (2.8)

$$\mathsf{P}_{II} = \frac{1}{2}\left(\frac{\omega_{3m}}{\alpha_3} + \frac{\omega_{1n}}{\alpha_1}\right)\,(\hat{N}_{m-n}^{31} - \hat{N}_{mn}^{31})(\alpha_m^{3i\dagger}\,\alpha_n^{1i\dagger} + \alpha_{-m}^{3j\dagger}\alpha_{-n}^{1j\dagger})\,. \tag{3.30}$$

At large μ

$$\frac{\omega_{3m}}{\alpha_3}+\frac{\omega_{1n}}{\alpha_1}=-\frac{1}{2\mu}\left(m^2-\frac{n^2}{y^2}\right)+\mathcal{O}\left(\frac{1}{\mu^3}\right), \tag{3.31}$$

we have

$$\mathsf{P}_{II}=-\frac{1}{4\mu}\left(m^2-\frac{n^2}{y^2}\right)(\hat{N}^{31}_{m-n}-\hat{N}^{31}_{mn})(\alpha^{3i\dagger}_m\alpha^{1i\dagger}_n+\alpha^{3j\dagger}_{-m}\alpha^{1j\dagger}_{-n}). \tag{3.32}$$

The first term in the prefactor gives

$$\mathsf{P}_I=-\frac{1}{\mu}\left(m^2-\frac{n^2}{y^2}\right). \tag{3.33}$$

Combining these expressions together with $C^{\rm vac}_{123}$ in eq. (2.6) and with the external states and the exponent eq. (A.15) we get the string theory answer eq. (2.13)

$$\frac{C^{\rm vac}_{123}}{\mu}\left(\frac{n^2}{y^2}-m^2\right)\left[\frac{1}{2}(\hat{N}^{31}_{m-n}-\hat{N}^{31}_{mn})+\hat{N}^{31}_{mn}\right]\hat{N}^{31}_{mn}=\frac{C^{\rm vac}_{123}}{2\mu}\left(\frac{n^2}{y^2}-m^2\right)(\hat{N}^{31}_{m-n}+\hat{N}^{31}_{mn})\hat{N}^{31}_{mn}. \tag{3.34}$$

Finally, using the expressions for the Neumann matrices from the appendix

$$(\hat{N}^{31}_{m-n}+\hat{N}^{31}_{mn})\hat{N}^{31}_{mn}=\frac{2}{\pi^2}\frac{\sin^2(\pi my)}{y(m^2-n^2/y^2)^2}\left(m^2+\frac{mn}{y}\right), \tag{3.35}$$

we derive the right hand side of the SYM expression eq. (3.29).

3.3.2 $\mathbf{j_n\, i_{-n} + vac \to i_m\, j_{-m}}$

The SYM result eq. (3.1) is

$$\mu(\Delta_1+\Delta_2-\Delta_3)C_{123}=-C^{\rm vac}_{123}\frac{\sin^2(\pi my)}{y\,\pi^2\mu}\frac{m^2-mn/y}{m^2-n^2/y^2}. \tag{3.36}$$

The string theory prediction is obtained as in the previous subsection, except that now P_{II} receives contributions from the other two oscillator bilinears,

$$\mathsf{P}_{II}=-\frac{1}{2}\left(\frac{\omega_{3m}}{\alpha_3}+\frac{\omega_{1n}}{\alpha_1}\right)(\hat{N}^{31}_{m-n}-\hat{N}^{31}_{mn})(\alpha^{3i\dagger}_m\alpha^{1i\dagger}_{-n}+\alpha^{3j\dagger}_{-m}\alpha^{1j\dagger}_n). \tag{3.37}$$

The net result is

$$\frac{C^{\rm vac}_{123}}{\mu}\left(\frac{n^2}{y^2}-m^2\right)\left[-\frac{1}{2}(\hat{N}^{31}_{m-n}-\hat{N}^{31}_{mn})+\hat{N}^{31}_{m-n}\right]\hat{N}^{31}_{m-n}, \tag{3.38}$$

which, using the expressions for the Neumann matrices at large μ, agrees precisely with the SYM expression eq. (3.36).

3.3.3 $\mathbf{j_n\, j_{-n} + vac \to i_m\, i_{-m}}$

The SYM prediction eq. (3.1) is

$$\mu(\Delta_1+\Delta_2-\Delta_3)C_{123}=\frac{1}{2}C^{\rm vac}_{123}\frac{\sin^2(\pi my)}{y\,\pi^2\mu}. \tag{3.39}$$

The external string states are ${}_{123}\langle 0|\alpha^{3i}_{m}\alpha^{3i}_{-m}\alpha^{1j}_{n}\alpha^{1j}_{-n}$ This is the flavour changing process and the leading order in $1/\mu$ contribution comes from P_{II} and is given by eq. (3.18). The exponent in eq. (A.15) gives the factor of

$$\hat{N}^{11}_{n-n}\alpha^{1j^\dagger}_{n}\alpha^{1j^\dagger}_{-n}=\frac{1}{\mu 4\pi y}\alpha^{1j^\dagger}_{n}\alpha^{1j^\dagger}_{-n}\,. \tag{3.40}$$

Putting this all together in eq. (2.13) we get the string theory prediction

$$\frac{1}{2}\,C^{\rm vac}_{123}\,\frac{\sin^2(\pi m y)}{y\,\pi^2\mu}\,, \tag{3.41}$$

which is precisely the SYM result eq. (3.39).

3.3.4 $\mathbf{i_n\,i_{-n}}$ + vac $\to$ $\mathbf{i_m\,i_{-m}}$

The SYM prediction eq. (3.1) is

$$\mu(\Delta_1+\Delta_2-\Delta_3)C_{123}=-\frac{1}{2}\,C^{\rm vac}_{123}\,\frac{\sin^2(\pi m y)}{y\,\pi^2\mu}\,\frac{3m^2+n^2/y^2}{m^2-n^2/y^2}\,. \tag{3.42}$$

It is easy to see that the corresponding string prediction is simply the sum of the string predictions of the three previous subsections. This is again in agreement with SYM since the right hand side of eq. (3.42) is equal to the sum of the right hand sides of equations eq. (3.29),eq. (3.36) and eq. (3.39).

3.4 Three impurities

Both of the processes considered below involve 2 supergravity states and are of the first class, consequently only the first term in the prefactor, P_I, gives a nontrivial contribution:

$$\mathsf{P}_I=\mu(\Delta_1+\Delta_2-\Delta_3)=-\frac{n_1^2+n_2^2+n_3^2}{\mu}\,, \tag{3.43}$$

where $n_1=-(n_2+n_3)$.

3.4.1 $\mathbf{1_0\,2_0\,3_0}$ + vac $\to$ $\mathbf{1_{n_1}\,2_{n_2}\,3_{n_3}}$

The external string state is:

$${}_{123}\langle 0|\alpha^{3i_1}_{n_1}\alpha^{3i_2}_{n_2}\alpha^{3i_3}_{n_3}\alpha^{1i_1}_{0}\alpha^{1i_2}_{0}\alpha^{1i_3}_{0} \tag{3.44}$$

and the relevant part of the exponent in eq. (A.15) gives

$$\hat{N}^{31}_{n_1 0}\,\hat{N}^{31}_{n_2 0}\,\hat{N}^{31}_{n_3 0}\,. \tag{3.45}$$

The resulting string prediction is

$$-\mu(\Delta_1+\Delta_2-\Delta_3)\,C^{\rm vac}_{123}\,\frac{\sin(\pi y n_1)\sin(\pi y n_2)\sin(\pi y n_3)}{y^{3/2}\pi^3\,n_1n_2n_3}\,. \tag{3.46}$$

which reproduces eq. (3.6) precisely.

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3.4.2 $\mathbf{1_0 2_0 + 3_0 \rightarrow 1_{n_1} 2_{n_2} 3_{n_3}}$

Here the external string state is:

$$
{}_{123}\langle 0| \alpha^{3i_1}_{n_1} \alpha^{3i_2}_{n_2} \alpha^{3i_3}_{n_3} \alpha^{1i_1}_{0} \alpha^{1i_2}_{0} \alpha^{2i_3}_{0} \tag{3.47}
$$

and the bosonic overlap, eq. (A.15), gives gives

$$
\hat{N}^{31}_{n_1 0} \hat{N}^{31}_{n_2 0} \hat{N}^{32}_{n_3 0} \,. \tag{3.48}
$$

The resulting string prediction is

$$
\mu(\Delta_1 + \Delta_2 - \Delta_3)\, C^{\rm vac}_{123} \frac{\sin(\pi y n_1) \sin(\pi y n_2) \sin(\pi y n_3)}{y\sqrt{1-y}\pi^3\, n_1 n_2 n_3} \,, \tag{3.49}
$$

which precisely reproduces eq. (3.7).

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A. Neumann matrices in the large-μ^2 perturbation theory

We first specify the notation and conventions used in pp-wave string field theory. The combination $\alpha' p^+$ for the r-th string is denoted $\alpha_{\rm r}$ and $\sum_{\rm r=1}^3 \alpha_{\rm r} = 0$. As is standard in the literature, we will choose a frame in which $\alpha_3 = -1$

$$
\alpha_{\rm r} = \alpha' p^+_{(\rm r)} \; : \qquad \alpha_3 = -1 \,, \qquad \alpha_1 = y \,, \qquad \alpha_2 = 1 - y \,. \tag{A.1}
$$

In terms of the U(1) R-charges of the BMN operators in the three-point function, $\langle \mathcal{O}_1^{J_1} \mathcal{O}_2^{J_2} \bar{\mathcal{O}}_3^{J} \rangle$, where $J = J_1 + J_2$, we have

$$
y = \frac{J_1}{J} \,, \qquad 1 - y = \frac{J_2}{J} \,, \qquad 0 < y < 1 \,. \tag{A.2}
$$

The effective SYM coupling constant eq. (2.2) in the frame eq. (A.1) takes a simple form

$$
\lambda' = \frac{1}{(\mu p^+ \alpha')^2} \equiv \frac{1}{(\mu \alpha_3)^2} = \frac{1}{\mu^2} \,. \tag{A.3}
$$

Here μ is the mass parameter which appears in the pp-wave metric, in the chosen frame it is dimensionless[9] and the expansion in powers of $1/\mu^2$ is equivalent to the perturbative expansion in λ'. Finally the frequencies are defined via,

$$
\omega_{\rm r m} = \sqrt{m^2 + (\mu \alpha_{\rm r})^2} \,. \tag{A.4}
$$

[9] It is $p^+\mu$ which is invariant under longitudinal boosts and is frame-independent.

The infinite-dimensional Neumann matrices, $N^{\rm rs}_{mn}$ are usually specified in the original a-oscillator basis of the string field theory. In this basis the bosonic overlap factor $|V_B\rangle$ of the 3-strings vertex is given by

$$|V_B\rangle = \exp(\frac{1}{2}\sum_{\rm r,s=1}^{3}\sum_{m,n=-\infty}^{\infty} a^{{\rm r}\,I^\dagger}_m N^{\rm rs}_{mn} a^{{\rm s}\,I^\dagger}_n)|0\rangle\,. \tag{A.5}$$

However, for the purposes of the pp-wave/SYM correspondence it is more convenient to use another, the so-called BMN or α-basis of string oscillators, which is in direct correspondence with the BMN operators in gauge theory. The two bases are related as follows:

$$\alpha_n = \frac{1}{\sqrt{2}}\left(a_{|n|} - i\,{\rm sign}(n) a_{-|n|}\right)\,, \qquad \alpha_0 = a_0\,, \tag{A.6}$$

and satisfy the same oscillator algebra

$$[\alpha_m, \alpha^\dagger_n] = \delta_{mn}\,. \tag{A.7}$$

In this basis, the bosonic overlap factor eq. (A.5) in the vertex reads

$$|V_B\rangle = \exp\left(\frac{1}{2}\sum_{\rm r,s=1}^{3}\sum_{m,n=-\infty}^{\infty} \alpha^{{\rm r}\,I^\dagger}_m \hat{N}^{\rm rs}_{mn} \alpha^{{\rm s}\,I^\dagger}_n\right)\,, \tag{A.8}$$

where $\hat{N}$ are the Neumann matrices in the α-basis and are related to the N's via (here $m,n>0$):

$$\hat{N}^{\rm rs}_{mn} = \hat{N}^{\rm rs}_{-m-n} := \frac{1}{2}(N^{\rm rs}_{mn} - N^{\rm rs}_{-m-n})\,, \tag{A.9}$$

$$\hat{N}^{\rm rs}_{m-n} = \hat{N}^{\rm rs}_{-mn} := \frac{1}{2}(N^{\rm rs}_{mn} + N^{\rm rs}_{-m-n})\,, \tag{A.10}$$

$$\hat{N}^{\rm rs}_{m0} = \hat{N}^{\rm rs}_{-m0} := \frac{1}{\sqrt{2}}N^{\rm rs}_{m0} = \hat{N}^{\rm rs}_{0m} = \hat{N}^{\rm rs}_{0-m}\,, \tag{A.11}$$

$$\hat{N}^{\rm rs}_{00} := N^{\rm rs}_{00}\,. \tag{A.12}$$

To derive these expressions we have equated eq. (A.5) and eq. (A.8), and used the known properties of the original perturbative Neumann matrices:

$$N^{\rm rs}_{MN} = N^{\rm sr}_{NM}\,, \qquad \text{for all } -\infty < M,N < +\infty\,, \tag{A.13}$$

$$N^{\rm rs}_{-m0} = 0\,, \qquad N^{\rm rs}_{m-n} = 0\,, \qquad \text{for } m,n>0\,. \tag{A.14}$$

Making use of eq. (A.11) and eq. (A.13), we can write eq. (A.8) as

$$|V_B\rangle = \exp\Bigg[\frac{1}{2}\sum_{\rm r,s=1}^{3}\sum_{m,n=1}^{\infty}\Big(\hat{N}^{\rm rs}_{00}\alpha^{{\rm r}\,I^\dagger}_0\alpha^{{\rm s}\,I^\dagger}_0 + 2\hat{N}^{\rm rs}_{m0}(\alpha^{{\rm r}\,I^\dagger}_m\alpha^{{\rm s}\,I^\dagger}_0 + \alpha^{{\rm r}\,I^\dagger}_{-m}\alpha^{{\rm s}\,I^\dagger}_0) + \tag{A.15}$$

$$+ \hat{N}^{\rm rs}_{mn}(\alpha^{{\rm r}\,I^\dagger}_m\alpha^{{\rm s}\,I^\dagger}_n + \alpha^{{\rm r}\,I^\dagger}_{-m}\alpha^{{\rm s}\,I^\dagger}_{-n}) + \hat{N}^{\rm rs}_{m-n}(\alpha^{{\rm r}\,I^\dagger}_m\alpha^{{\rm s}\,I^\dagger}_{-n} + \alpha^{{\rm r}\,I^\dagger}_{-m}\alpha^{{\rm s}\,I^\dagger}_n)\Big)\Bigg]\,.$$

We now present the explicit expressions for the Neumann matrices in the original a-basis obtained by expanding the results of [29] in powers of $1/\mu^2$. These expressions are needed for calculations in section 3.

$$
\begin{aligned}
N^{31}_{mn} &= \frac{2(-1)^{m+n+1}}{\pi}\frac{m\sin(\pi m y)}{\sqrt{y}(m^2-n^2/y^2)} + \mathcal{O}\left(\frac{1}{\mu^2}\right),\\
N^{32}_{mn} &= \frac{2(-1)^{m}}{\pi}\frac{m\sin(\pi m y)}{\sqrt{1-y}(m^2-n^2/(1-y)^2)} + \mathcal{O}\left(\frac{1}{\mu^2}\right),\\
N^{21}_{mn} &= \frac{1}{\mu}\frac{(-1)^{n+1}}{2\pi}\frac{1}{\sqrt{y(1-y)}} + \mathcal{O}\left(\frac{1}{\mu^3}\right),\\
N^{33}_{mn} &= \mathcal{O}\left(\frac{1}{\mu^3}\right),\\
N^{11}_{mn} &= \frac{1}{\mu}\frac{(-1)^{m+n}}{2\pi}\frac{1}{y} + \mathcal{O}\left(\frac{1}{\mu^3}\right),\\
N^{22}_{mn} &= \frac{1}{\mu}\frac{1}{2\pi}\frac{1}{1-y} + \mathcal{O}\left(\frac{1}{\mu^3}\right)
\end{aligned}
\tag{A.16}
$$

$$
\begin{aligned}
N^{31}_{-m-n} &= \frac{2(-1)^{m+n}}{\pi}\frac{n\sin(\pi m y)}{y^{3/2}(m^2-n^2/y^2)} + \mathcal{O}\left(\frac{1}{\mu^2}\right),\\
N^{32}_{-m-n} &= \frac{2(-1)^{m+1}}{\pi}\frac{n\sin(\pi m y)}{(1-y)^{3/2}(m^2-n^2/(1-y)^2)} + \mathcal{O}\left(\frac{1}{\mu^2}\right),\\
N^{21}_{-m-n} &= \mathcal{O}\left(\frac{1}{\mu^3}\right),\\
N^{33}_{-m-n} &= \frac{1}{\mu}\frac{2(-1)^{m+n}}{\pi}\sin(\pi m y)\sin(\pi n y) + \mathcal{O}\left(\frac{1}{\mu^3}\right),\\
N^{11}_{-m-n} &= \mathcal{O}\left(\frac{1}{\mu^3}\right),\\
N^{22}_{-m-n} &= \mathcal{O}\left(\frac{1}{\mu^3}\right)
\end{aligned}
\tag{A.17}
$$

$$
\begin{aligned}
N^{33}_{00} &= 0\,, \qquad N^{31}_{00} = -\sqrt{y}\,, \qquad N^{32}_{00} = -\sqrt{1-y}\,,\\
N^{12}_{00} &= \frac{1}{\mu}\frac{(-1)}{4\pi}\frac{1}{\sqrt{y(1-y)}} = N^{21}_{00}\,,\\
N^{11}_{00} &= \frac{1}{\mu}\frac{1}{4\pi}\frac{1}{y}\,,\\
N^{22}_{00} &= \frac{1}{\mu}\frac{1}{4\pi}\frac{1}{1-y}\,.
\end{aligned}
\tag{A.18}
$$

For the zero-positive Neumann matrices we have

$$N^{31}_{0n} = 0\,, \qquad N^{32}_{0n} = 0\,, \qquad N^{33}_{0n} = 0 \tag{A.19}$$

$$\begin{aligned}
N^{13}_{0n} &= \frac{\sqrt{2}(-1)^{n+1}}{\pi}\frac{\sin(\pi n y)}{n\sqrt{y}} + \mathcal{O}\left(\frac{1}{\mu^2}\right),\\
N^{23}_{0n} &= \frac{\sqrt{2}(-1)^{n}}{\pi}\frac{\sin(\pi n y)}{n\sqrt{1-y}} + \mathcal{O}\left(\frac{1}{\mu^2}\right),\\
N^{21}_{0n} &= \frac{1}{\mu}\frac{\sqrt{2}(-1)^{n+1}}{4\pi}\frac{1}{\sqrt{y(1-y)}} + \mathcal{O}\left(\frac{1}{\mu^3}\right),\\
N^{12}_{0n} &= -\frac{1}{\mu}\frac{\sqrt{2}}{4\pi}\frac{1}{\sqrt{y(1-y)}} + \mathcal{O}\left(\frac{1}{\mu^3}\right),\\
N^{11}_{0n} &= \frac{1}{\mu}\frac{\sqrt{2}(-1)^{n}}{4\pi}\frac{1}{y} + \mathcal{O}\left(\frac{1}{\mu^3}\right),\\
N^{22}_{0n} &= \frac{1}{\mu}\frac{\sqrt{2}}{4\pi}\frac{1}{1-y} + \mathcal{O}\left(\frac{1}{\mu^3}\right).
\end{aligned} \tag{A.20}$$

The behavior of Neumann matrices at $\mu \to \infty$ was first analyzed in [3] by resumming all-orders power expansions in the large μ (small λ') limit. In manipulating with multiplications of infinite dimensional matrices the authors of [3] encountered divergences which were regularized using the zeta-function regularization. Recently in [29] the Neumann matrices at $\mu \to \infty$ were calculated using a different method leading to manifestly regular expressions. The results of [29] which we use in this paper agree with the expressions obtained [3] at order $(1/\mu)^0$, but not at higher orders in $1/\mu$. Two comments are in order:

1. These perturbative expressions for the Neumann matrices should not be interpolated to the flat space expressions at $\mu = 0$ since essential singularities at $\mu = \infty$ were discarded.

2. Each of the Neumann matrices is expanded in powers of $1/\mu^2$, the odd powers of μ can appear only as an overall multiplicative factor. Hence the fractional powers of λ' hopefully should not appear in the string theory prediction at higher orders, thus making a happy connection with the gauge theory interpretation.

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